

# Geometrical Analysis of the Triple Leaf Pattern in Metropolis

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## Abstract

In this paper, the triple leaf pattern located in the floor mosaics of the Reception Hall in the ancient city of Metropolis has been analyzed geometrically. It has been determined that the triple leaf pattern considered as a floral pattern was derived from the solution of a problem by means of geometry. In addition, by comparing the abstract state and the actual state of the pattern, the effect of practical necessities arising from the material and workmanship on the pattern has been evaluated. This standard drawing obtained geometrically reveals a stylistic development of the pattern, from the regular leaf pattern to the twisted leaf figure. It has been thought that geometrical studies on mosaics could contribute to studies about the identification of local workshops in Roman period and to conservation practices.

## Özet

Bu çalışmada, Metropolis antik kentinde Resepsiyon Salonu taban mozaiklerinde yer alan üçlü yaprak motifi geometrik olarak analiz edilmiştir. Bitkisel bir figür olarak düşünülen üçlü yaprak motifinin bir geometri probleminin çözümünden türetildiği tespit edilmiştir. Ayrıca, motifin soyut hali ve gerçek durumu karşılaştırılarak materyal ve işçilikten kaynaklanan gerekliliklerin motife etkileri değerlendirilmiştir. Geometrik olarak elde edilen bu standart çizim, düzgün yaprak motifinden bükülmüş yaprak motifine doğru, motifin üslup gelişimini açıklamaktadır. Mozaikler üzerinde geometri çalışmalarının Roma dönemi yerel atölyelerin kimliklendirilmesiyle ilgili çalışmalara ve konservasyon uygulamalarına katkı sağlayabileceği düşünülmektedir.

## Introduction

The triple leaf patterns of the floor mosaics in the so-called Reception Hall in the ancient city of Metropolis (fig. 1) have been chosen as the subject of this paper. The aforementioned mosaic panel contains geometric patterns, human figures, bird and fish figures, and triple leaf patterns. The panel frame and the included geometric patterns were analyzed geometrically. However, the floral figures and human figures in the panel were left out for a separate study, and not included in this analysis. The analysis demonstrated that the panel was created by repeating a simple element pattern in a certain order, and that the complicated geometric patterns are the result of this repetition.<sup>2</sup>



Fig. 1: Mask Mosaic from the Reception Hall.

As it is understood that there is a close relationship between the design of the panel and the knowledge of geometry, the question of whether the triple leaf pattern is also a product of geometric design has arisen. For this reason, the geometric analysis of the triple leaf pattern has been chosen as the principal subject of this study. Another aim of this study is to contribute to the exploration and exemplification of geometry for examining floral patterns, identifying local styles, characterizing common patterns, and to explain the stylistic development of the figures.

The framework of the method used in this analysis involved examining the geometric applications which produce smooth leaf shape, solving the twisting phenomenon, and obtaining the abstract model of the twisted triple leaf pattern. Then, it compared the differences between the smooth leaf model and the twisted leaf pattern to reveal the interaction between the abstract and real state of the pattern. As a result, the analysis determined the effect of practical necessities arising from the material and workmanship.

### **The Mask Mosaic in Metropolis**

Metropolis was an important ancient city located in the center of the Western Anatolian coast (named Ionia in the ancient period) between Smyrna and Ephesus. The quality of its ruins and finds revealed during recent excavations shows that it was just as important as the rest of the cities in the same area. The city's real establishment and development

took place in the Hellenistic period as it was influenced by the Kingdom of Pergamum and enhanced significantly during the Roman period.<sup>3</sup> Although there are technical and stylistic differences in each of these mosaics, it can be generally assumed that the mosaic tradition in Metropolis was enriched during the 3<sup>rd</sup> and 4<sup>th</sup> centuries A.D.<sup>4</sup>

The building identified as the Reception Hall of the Theater is named because of its theatrical symbols.<sup>5</sup> It is located on the eastern side of the Theatre and its floor is decorated with panel mosaics. In fact, the mosaic floor consists of two different panels, one in the center and one on the eastern border. The eastern panel (measuring 2,02 × 3,48 m) is important since it conveys an impression of a Reception Hall. Eight of the eleven figures on this panel consist of bird and fish depictions, while the remaining three figures are theater masks located in the center of the panel. In this way, the aforementioned symbols are related to both the theater and the banquet. The annexes, which functioned as a cellar or a kitchen, support the function of the building. Similar examples of this type of building are found in the Terrace House of Ephesus,<sup>6</sup> and Bau Z in Pergamum.<sup>7</sup>

The middle panel is surrounded by a frame consisting of an intertwined square measuring 2,70 × 3,78 m, which is divided into six equal parts. There is one part in the southeastern area which is not preserved, but the other five parts reveal a workmanship of very high quality with figural mosaics.<sup>8</sup> Many different ideas and suggestions have been presented for the interpretation regarding the motif and the dating of the five figures on the middle panel. According to Recep Meriç, who was the first person to excavate this place, the panels have figures relating to Dionysus<sup>9</sup> or the Four Seasons<sup>10</sup> and can be dated back to the 2<sup>nd</sup>–3<sup>rd</sup> century A.D.

## **Geometric Analysis and Comparison**

### **Preliminary Assessments, Hypothesis and Method**

The mosaic panel contains geometric patterns, human, bird and fish figures, and triple leaf patterns. The panel frame and the geometric patterns have been analyzed geometrically. It has been understood that the panel was created by repeating the element pattern within a certain order, and that the complicated geometric patterns are the result of this repetition.<sup>11</sup> It is obvious that the geometric patterns are the result of the geometric planning. However, the repeated use of plant motifs in the dominant design of geometric planning leads to the question of whether plant figures are the subject of geometric planning.

The triple leaf patterns were placed in semi-octagonal spaces between the geometric patterns (fig. 1). So, the dimensions of the patterns are related to the dimensions of the area allocated to them. On the other hand, just as with the repeating of the element pattern, the triple leaf motifs are also repeated, as if they were produced from a single copy. The preservation of similarity in repetition

indicates that a geometric model was used to make these motifs. For this reason, the hypothesis of this study is the proposition that the design process of the triple leaf pattern utilized a geometric model.

The motifs show deformations due to workmanship and the material used, and the leaf tips are twisted clockwise or counterclockwise. In the determination of the twisting direction, the concern for symmetry is evident. As an exception, it seems that all the leaves are twisted in the same direction in one figure. Therefore, it is understood that the twisting direction could be consciously preferred. If a model had been used in the design of the triple leaf pattern, this model should have features that allow differentiation in the twisting of the leaf tips.

While a regular leaf model is simple, the twisted leaf model is complex. In order to understand the difference between a regular leaf and a twisted leaf, it is necessary to examine geometric applications that produce the regular leaf pattern, to solve the twisting phenomenon, and to obtain the abstract model of the twisted leaf pattern. By comparing the abstract model of the pattern and the actual pattern, the relationships between the theoretical model and the practice can be revealed.

### **Six Leaves Pattern, Twisting Procedure, and Construction of the Twisted Triple Leaf Pattern**

To draw a regular hexagon on a circle, it is required to make six points on the circle with equal distance between them. The positions of these six points on the circle must be detected. A certain center point, a certain radius, and a compass are sufficient for this operation. Let the center point be  $O$ , the radius be  $r$ , and draw a circle by opening the compass' legs up to  $r$ . This circle can be named the main circle. Place the fixed end of the compasses on the main circle without altering the openness of the compasses, and draw a new circle. This circle cuts the main circle at two points. Draw new circles that take these intersection points as their centers. Every single circle that is drawn on the circumference of the main circle cuts the main circle at two points. There are seven circles in total, and their diameters are equal. There are six points of intersection. These points can be marked as  $T_1, T_2, T_3, T_4, T_5, T_6$ . Thus, the distances between successive pairs of points are equal. These six points are the corners of the hexagon (fig. 2).

There are two equilateral triangles, the triangle  $T_1T_3T_5$  and the triangle  $T_2T_4T_6$ . These triangles intersect. These equilateral triangles form a hexagram. Intersecting arches, which are the traces left behind by the compass during the drawing process, constitute a six leaf pattern called a rosette in mosaics (fig. 2). This pattern consists of six regular leaf patterns.

Straight lines, which connect the determined six points on the main circle, constitute a hexagram, a hexagon, two equilateral triangles, and six central beam lines. The intersection points  $A, B, C,$  and  $D$  are the centers of the circles, which are used to convert the floral pattern into a triple leaf pattern. Every leaf of the floral

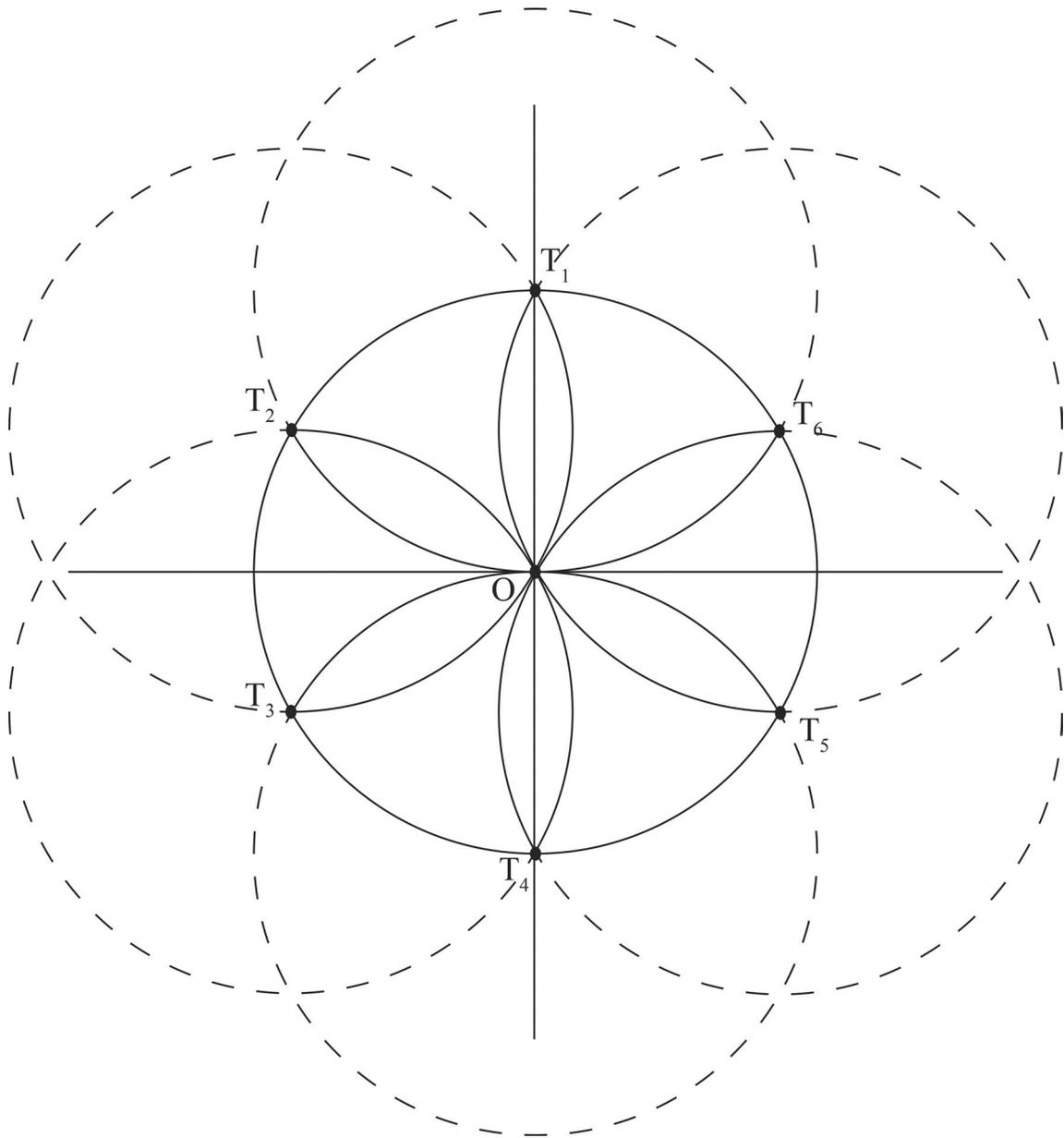


Fig. 2: Geometric Drawing of the Six Leaf or Rosette Pattern.

pattern will bend smoothly from point X. The obtained pattern is the twisted triple leaf pattern (fig. 3).

**Greek Mathematics and the Abstract Model of the Triple Leaf Pattern**

The problem known as trisection of an angle by using only a measureless ruler and a compass is a famous geometry problem of the ancient period.<sup>12</sup> This problem is about

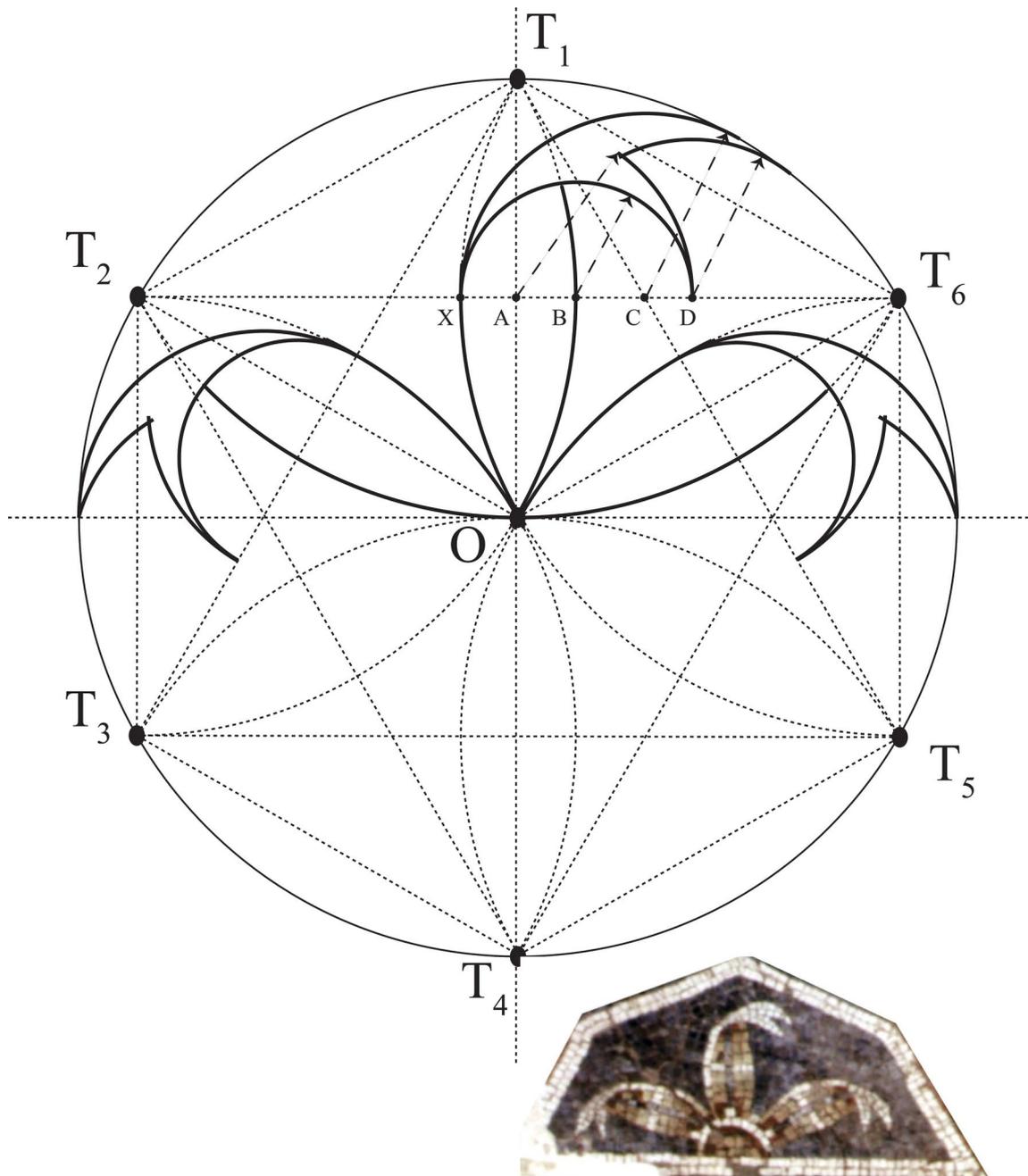


Fig. 3: Geometric Drawing of the Triple Leaf Pattern.

determining the drawing facilities of a ruler and compass. The model used here is the model that proves that the hexagon can be drawn properly using a measureless ruler and compasses. This model represents a regular six leaf pattern. For deriving the twisted triple leaf pattern from the regular six leaf pattern, natural intersection points have been used in the model to determine the diameters of those circles used in the twisting

operation. Namely, it is understood that natural rates were used in the design process, and that there were no numerical calculations. A compass and a measureless ruler are enough for the design.

For a perfect twist, the arch representing the leaf contour must be properly spliced into the arch representing the twisting. At this point, it is notable that another important geometric knowledge is also needed for the twisting operation: for splicing the arcs of two circles, the centers of two circles and the contact point of the arches must be on the same straight line.<sup>13</sup> It is seen that this rule was used professionally in the twisting procedure.

### **Comparison of Abstract Model and Real Pattern**

Operations of the triple leaf pattern and the size of the pattern depend on the radius of the main circle on which the triple leaf pattern is built. This radius depends on the radius of the space allocated to the pattern in the geometric frame. The triple leaf pattern may be protected from deformation by selecting an appropriate radius, or by sliding the figure to fit the space, even if the allocated area is deformed during the tessellation process. It is seen that the semi-hexagonal areas have been deformed, and that the positions and the shapes of the semi-circle cores of the figure have been slightly shifted.

The center of a triple leaf pattern is determined geometrically. According to the theoretical model, leaves must start from the center and extend outward. However, practically, because of the size of the tesserae, it is physically impossible that tessellation starts from the center of the pattern. Placing a semi-circular core on the heart of a triple leaf pattern is a good solution to save the day. In the theoretical model, leaf ends are gradually narrowing. Narrowed zones cannot be laid with tesserae after a certain stage. It is seen that the edge areas have been tessellated to the extent possible, sometimes with a single row of light stones.

It is seen that dark stones were laid vertically along the middle vein of the leaves. The vein surface and vein-free surface, or inner and outer part of the leaf, have also been distinguished by this dark line. This dark colored line is the vertical symmetry axis of the leaf in the geometric model. On the inside of the leaf, there is a horizontal line consisting of light colored tesserae. This horizontal line is the horizontal symmetry axis of the leaf in the geometric model. The interior of the leaves were depicted in dark color, and the exterior surface was tessellated with light colored stones. In the geometric model, the inner and the outer surface are visually distinguishable. It is understood that the geometric model also plays a guiding role for light-shadow separation, color preferences, and artistic operations.

Triple leaf patterns were placed symmetrically on the opposite sides of the main frame. The leaves were bent to different directions, except for one figure. This situation indicates that the twisting procedure was well known and skillfully used, thanks to a geometric model that was open to preferences regarding the bending direction. The results support the hypothesis.

### Conclusion

The abstract model of the triple leaf motif considered as a plant figure was obtained and a standard drawing method was determined. The abstract model of the pattern overlaps with the drawing method used in the proof procedure related to one of the famous problems of ancient Greek mathematics. It has been shown that this figure was derived from the pattern known as a rosette, by applying the twisting phenomenon in accordance with geometric rules. The twisting phenomenon provides an analytical explanation for the stylistic development of the floral patterns, and represents an example of the transition from a two-dimensional figure to a three-dimensional figure.

This study has revealed that the abstract model was used as a guide in the color transitions, in determining the separation lines between light-shadow zones, in the laying of tesserae lines, in determining the figure dimensions and boundaries, and in similar artistic operations. It has been seen that the size of the tesserae is an effective physical factor in the application process, and that there are differentiations from the abstract model in the ever-narrowing regions of the figure due to the effect of tesserae size. That compensatory maneuvers were used in the junction areas of the narrowing regions has been observed.

If the information conveyed by Vitruvius, that mosaicists were using only a ruler, compasses, and spirit level<sup>14</sup> is also taken into account, it is obvious that the studies about geometry influenced mosaicists both in theory and in practice. Therefore, it is evaluated that geometric studies on mosaics can contribute to the examination of floral patterns and also human figures,<sup>15</sup> to identify local styles, to characterize common patterns, and to explain the stylistic development of the figures. In addition, it has been thought that geometric studies on mosaics could contribute to studies concerning the identification of local workshops in Roman period and to conservation practices.

### Notes

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<sup>2</sup> Öz – Aydoğdu 2016.

<sup>3</sup> Aybek et al. 2009, 39.

<sup>4</sup> Öz 2012a, 147.

<sup>5</sup> Meriç 1999, 336.

<sup>6</sup> Lang-Auinger 1996, 205.

<sup>7</sup> Radt 1988, 102.

<sup>8</sup> Öz 2012b, 703.

<sup>9</sup> Meriç 2004, 99.

<sup>10</sup> Parrish 2007, 19.

<sup>11</sup> Öz – Aydoğdu 2016.

<sup>12</sup> Heath 1921, 235–244.

<sup>13</sup> Euclid 2.1–77.

<sup>14</sup> Vitruvius. De arch. 7.1.3.

<sup>15</sup> Aydoğdu 2015.

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