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Probabilistic calibration of radiocarbon dates with specific examples from Northeastern Africa

Introduction

Radiocarbon dating relies on the assumption that the biospheric inventory of C-14 has remained constant during the past 100,000 years. This assumption was tested 40 years ago by Arnold and Libby (1949: 678) with the accuracy of *ca.* 10% by dating known-age Egyptian samples. However, with the improvement of the accuracy it was realized that this assumption is not precisely true. Systematic studies of discrepancies between C-14 and calendric dates, based on accurate C-14 determinations in dendrochronologically dated tree-ring samples have led to publication of numerous versions of calibration curves and tables (Suess 1970; 1979; Damon *et al.* 1974; Ralph *et al.* 1973; Switsur 1973; Clark 1975), but the real breakthrough in the calibration was achieved with publication of the "Calibration Issue" of *Radiocarbon*, with three high-precision calibration curves by Stuiver and Pearson (1986: 805), Pearson and Stuiver (1986: 839) and Pearson *et al.* (1986: 911).

Practical application of those high-precision calibration curves is, however, not simple, and the interpretation of obtained calendric ages is not straightforward. Because of numerous variations in calibration curves the correspondence between conventional C-14 dates and calendric ages is not exact, and, as a rule, there are several values of calendric ages corresponding to a given C-14 date. For example, calendric age corresponding to C-14 date 3,600 B.P. may be easily read as equal to 1,970 B.C., but for C-14 date 3,500 B.P. we obtain five values of calendric age, equal to 1,780, 1,795, 1,820, 1,835 and 1,880 B.C. Similarly, if the error of conventional C-14 date is taken into account, we have problem of multiple intervals. For example, for C-14 date $4,200 \pm 50$ B.P., we obtain three intervals of calendric age: 2,700 - 2,725 B.C., 2,770 - 2,810 B.C. and 2,865 - 2,890 B.C.

In order to overcome the difficulties caused by multiple intercepts with calibration curve we have introduced the concept of probabilistic calibration of radiocarbon dates and developed a set of appropriate computer procedures. The idea of the probabilistic calibration consists of transforming initial probability distribution of conventional C-14 date into calendric time scale and selecting appropriate parameters of resulting probability distribution as the measures of calendric age and its uncertainty.

Description of the computer procedure

The idea of probabilistic calibration was first introduced by Robinson (1985) and applied by Hassan and Robinson (1986) to calibration of a series of dates from Egypt, Nubia and Mesopotamia. The critique of this approach (Michczyńska *et al.* 1990) has led to more strict mathematical formulation of the algorithm of calibration, and the first version of calibration procedure was presented during the 2nd Symposium "Archaeology and C-14" in Groningen, September 1987, and an improved version was presented during the 13th International Radiocarbon Conference in Dubrovnik (Pazdur and Michczyńska 1989; *cf.* also Aitchison *et al.* 1989).

The system of calibration procedures was designed taking into account the specific tasks of archaeological application and includes three main options:

1. calibration of single date;
2. calibration of a set of arbitrary dates, representing same or different cultures/phases/objects;
3. calibration of a set of related dates obtained from a series of samples representing well-defined culture or phase.

Input data include: sample identifier (laboratory code and number, conv. B.P. date and its error); calibration output is presented on screen in form of graphs and numeric data. By pressing special function key <Prt Sc> it is possible to obtain hard copy of the screen on printer. In options 2 and 3 input data can be entered from diskette file, and also the screen output can be saved in a form of separate file and than retrieved at any time. Printed report including list of conventional and calibrated dates is also available.

Calibration is performed according to recently published high-precision calibration curves of Stuiver and Pearson (1986), Pearson and Stuiver (1986) and Pearson *et al.* (1986); range of conventional C-14 dates extends back to 6,210 B.P. Number of dates in series of input data for options 2 and 3 is not limited. Results can be presented in either B.P. or A.D./B.C. scale.

Examples

The possibilities offered by the development set of computer procedures and some difficulties involved in interpretation of results of calibration will be illustrated by several examples of C-14 dates in relation to NE Africa. Some dates are

taken from the volume edited by L. Krzyżaniak and M. Kobusiewicz (1984), other are more recent results obtained in the Gliwice Radiocarbon Laboratory. First we will present problems connected with calibration of single C-14 dates. Two specific cases are selected for this purpose, first we consider calibration of medium-accuracy C-14 dates, (*i.e.* C-14 dates quoted with error of ± 50 to ± 100 yr), which, in general, do not cause significant interpretational difficulties. Second example presents the calibration of high-accuracy C-14 date (dating error of *ca.* 30 yr or less), and illustrates typical difficulties with interpretation of high-accuracy dates.

The following examples consider calibration of groups of C-14 dates, obtained on same or different sites or cultures, and show several ways of presentation of results of C-14 dating in terms of calendric ages.

Calibration of single medium-accuracy date from El Tarif

As an example we will consider a date obtained on charcoal sample from Nagadian level in El Tarif site: (Gd-689: 5,070 \pm 60 B.P.), quoted by Ginter and Kozłowski (1984: 255). The results are presented in Fig. 1 and 2. Fig. 1 is a hard copy of the first screen of calibration output. Relevant input data (site and sample name, laboratory code and number, conventional B.P. date with 1 σ error, range of calibration) are listed at the left-hand side. Plots show probability distribution of conventional C-14 date (upper left-hand side), appropriate part of calibration curve (upper right-hand side) and resulting probability distribution of calendric age (lower plot). Negative values of calendric age denote B.C.

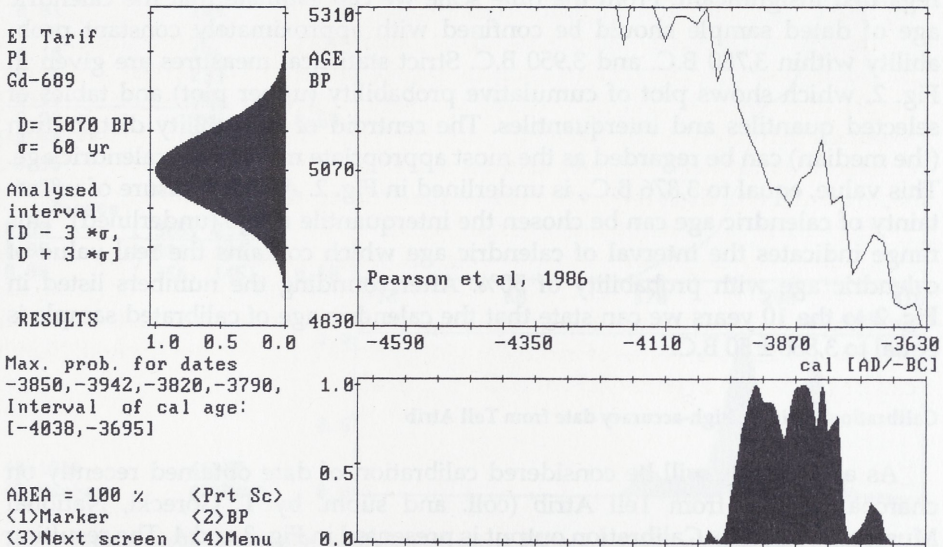


Fig. 1. Typical result obtained in calibration of single medium-precision date, part one (copy of the first screen of calibration output).

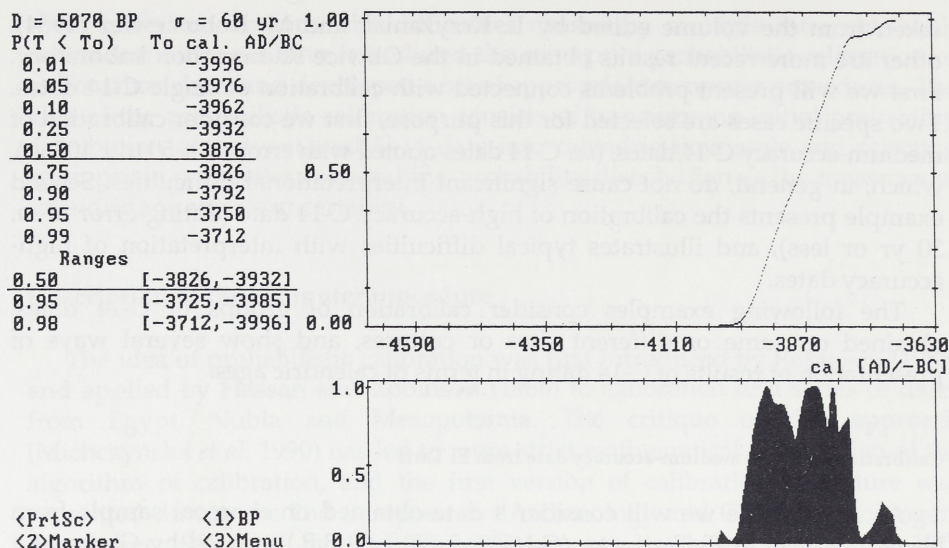


Fig. 2. Typical result obtained in calibration of single medium-precision date, part one (copy of the second screen of calibration output). Underlined are values of median and interquartile range.

dates. Because of wiggled shape of calibration curve in the considered interval of conventional C-14 dates (4,890 - 5,250 B.P.) also the shape of resulting probability distribution of calendric age shows several peaks of approximately same height, with two small peaks at the tails of probability distribution, which can be regarded insignificant. From the time scale we can estimate that the calendric age of dated sample should be confined with approximately constant probability within 3,780 B.C. and 3,950 B.C. Strict statistical measures are given in Fig. 2, which shows plot of cumulative probability (upper plot) and tables of selected quantiles and interquantiles. The centroid of probability distribution (the median) can be regarded as the most appropriate measure of calendric age. This value, equal to 3,876 B.C., is underlined in Fig. 2. As the measure of uncertainty of calendric age can be chosen the interquartile range (underlined). This range indicates the interval of calendric age which contains the real value of calendric age with probability of 50%. After rounding the numbers listed in Fig. 2 to the 10 years we can state that the calendric age of calibrated sample is equal to $3,880 \pm 50$ B.C.

Calibration of single high-accuracy date from Tell Atrib

As an example will be considered calibration of date obtained recently on charcoal sample from Tell Atrib (coll. and subm. by T. Górecki, National Museum, Warsaw). Calibration output is presented in Fig. 3 and 4. The resulting probability distribution of cal age reveals the presence of two pronounced peaks of approximately the same height. Their exact location on the calendric time

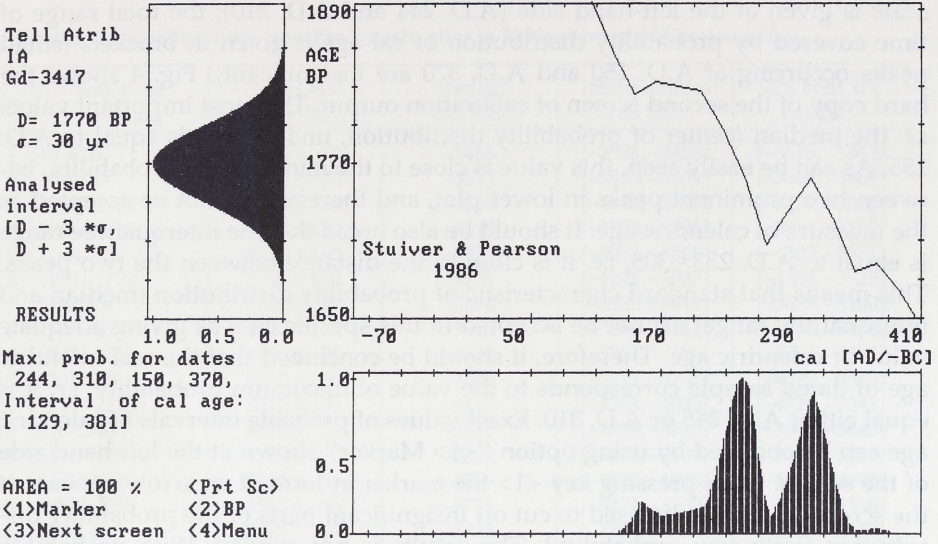


Fig. 3. Typical result obtained in calibration of single high-precision date, part one (copy of the first screen of calibration output).

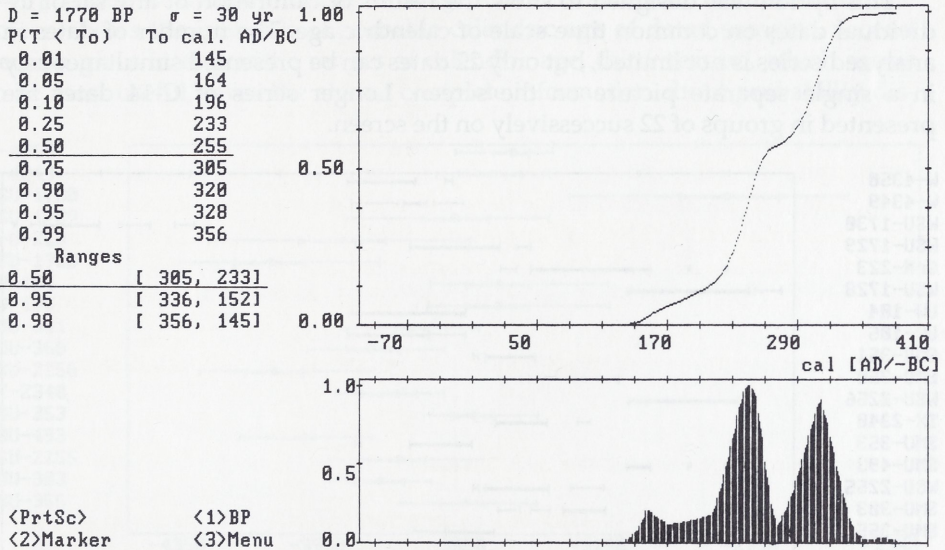


Fig. 4. Typical result obtained in calibration of single high-precision date, part two (copy of the second screen of calibration output). Underlined values (cf. Fig. 2) cannot be regarded as appropriate measures of calendric age and its uncertainty.

scale is given at the left-hand side (A.D. 244 and A.D. 310); the total range of time covered by probability distribution of cal age is given in brackets (small peaks occurring at A.D. 150 and A.D. 370 are insignificant). Fig. 4 shows the hard copy of the second screen of calibration output. The most important value, *i.e.* the median (center of probability distribution, underlined) is equal to A.D. 255. As can be easily seen, this value is close to the minimum of probability, between two prominent peaks in lower plot, and therefore cannot be accepted as the measure of calendric age. It should be also noted that the interquartile range is equal to A.D. 233 - 305, *i.e.* it is close to the distance between the two peaks. This means that standard characteristic of probability distribution (median and interquartile range) cannot be accepted in this specific case as giving adequate value of calendric age. Therefore, it should be concluded that the real calendric age of dated sample corresponds to the value of maximum probability, *i.e.* it is equal either A.D. 245 or A.D. 310. Exact values of probable intervals of calendric age can be obtained by using option "<1> Marker" shown at the left-hand side of the screen. After pressing key <1> the marker in form of an arrow appears at the screen and it can be used to cut off insignificant parts of the probability distribution (with low probability). The results is not explicit. With probability equal to *ca.* 65% the calendric age of dated charcoals is either A.D. 245+20-15 or A.D. 310+20-15.

Calibration of a set of arbitrary dates

This option was designed to show the results of calibration of any set of individual dates on common time scale of calendric age. The number of dates in analyzed series is not limited, but only 22 dates can be presented simultaneously in a single separate picture on the screen. Longer series of C-14 dates are presented in groups of 22 successively on the screen.

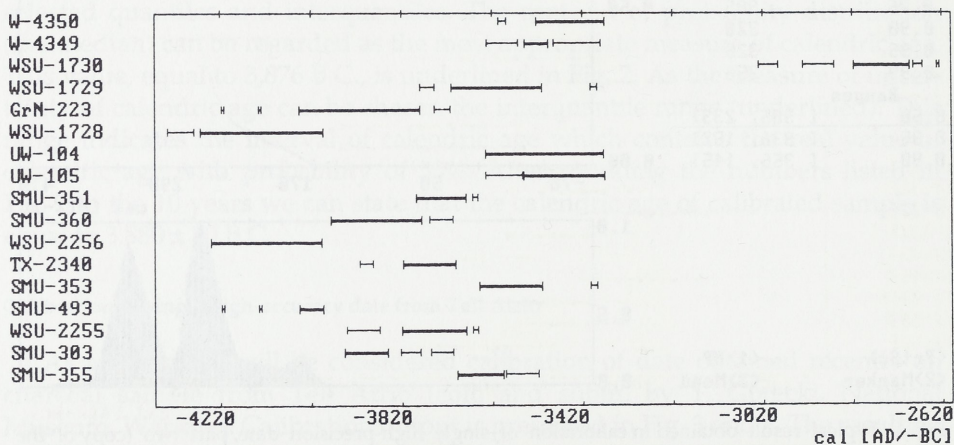


Fig. 5. Bar plot resulting from "cut-out" calibration performed on set of C-14 dates from El Khattara and Hierakonpolis (Hays 1984: 214). Intervals with maximum probability are marked as bold lines.

Table 1

Calibrated radiocarbon dates from Predynastic sites in El Khattara and Hierakonpolis.

No.	Lab. code and no.	Age C-14 conv. B.P.	Median cal. B.C.	Interquantiles cal. B.C.	95% conf. int. cal. B.C.
1	W-4350	4680 ± 60	-3458	[-3508, -3408]	[-3615, -3343]
2	W-4349	4730 ± 70	-3512	[-3585, -3431]	[-3667, -3369]
3	WSU-1730	4250 ± 130	-2858	[-2989, -2716]	[-3296, -2525]
4	WSU-1729	4830 ± 120	-3617	[-3705, -3521]	[-3919, -3357]
5	GrN-223	5110 ± 160	-3925	[-4053, -3808]	[-4302, -3571]
6	WSU-1728	5290 ± 130	-4131	[-4229, -4037]	[-4386, -3825]
7	UW-104	4720 ± 94	-3498	[-3583, -3414]	[-3688, -3190]
8	UW-105	4717 ± 94	-3495	[-3582, -3412]	[-3690, -3186]
9	SMU-351	4930 ± 70	-3738	[-3783, -3689]	[-3931, -3552]
10	SMU-360	5030 ± 100	-3847	[-3918, -3767]	[-4029, -3646]
11	WSU-2256	5270 ± 100	-4117	[-4203, -4039]	[-4328, -3841]
12	TX-2340	4970 ± 70	-3778	[-3876, -3725]	[-3949, -3645]
13	SMU-353	4780 ± 70	-3564	[-3621, -3509]	[-3705, -3384]
14	SMU-493	5214 ± 54	-4036	[-4122, -4005]	[-4217, -3848]
15	WSU-2255	4960 ± 100	-3771	[-3874, -3702]	[-3968, -3538]
16	SMU-303	5005 ± 69	-3823	[-3898, -3757]	[-3964, -3682]
17	SMU-355	4810 ± 80	-3592	[-3657, -3531]	[-3770, -3387]

Conventional C-14 dates see Hays 1984: 214, Table 1.

Three versions of presentation of calibration output are available in this option. First version shows the so-called "cut-out" calibration, *i.e.* the intervals of calendric age which are cut out of calibration curves by bands of width $[D-\sigma, D+\sigma]$, where D is conventional C-14 age and σ is its error. The results on the screen shows the intervals of probable calendric age of dated samples. Second version gives interquantile ranges of calendric age, third version, which seems to be most useful, gives the almost complete information about probability dis-

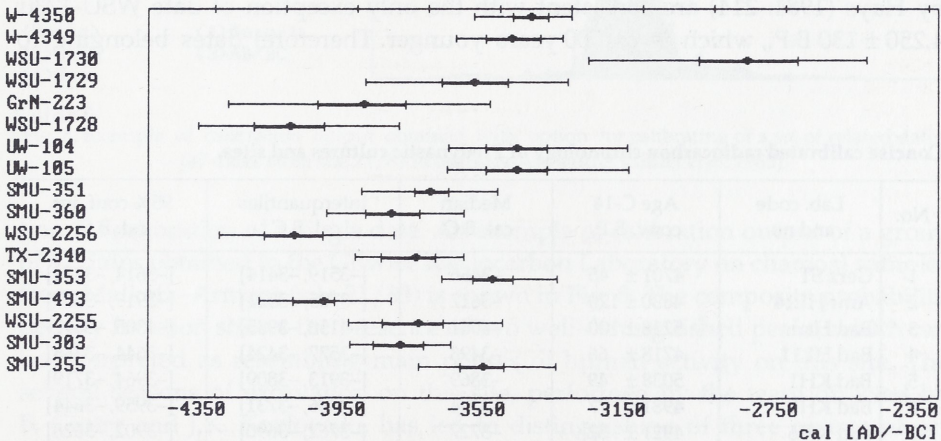


Fig. 6. Results of calibration of same data set as in Fig. 5 (El Khattara and Hierakonpolis; Hays 1984: 214) showing median (dots), interquantile ranges (bold lines) and 95% confidence intervals of calendric age.

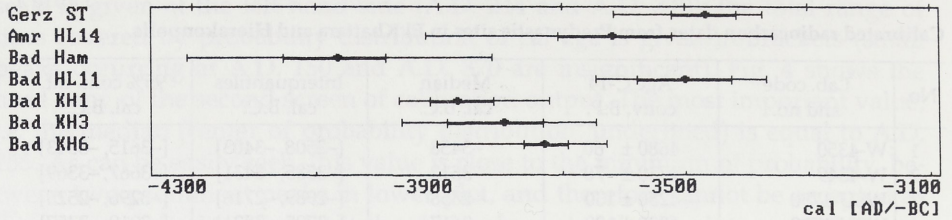


Fig. 7. Results of calibration of mean values of C-14 dates from indicated cultures and localities (after Hays 1984: 214);

Gerz: Gerzean; Amr: Amratian; Bad: Badarian;

ST: South Town; HL14: Hierakonpolis Loc. 14; Ham: Hamamiya; HL11: Hierakonpolis Loc. 11; KH: Khattara.

tributions of calendric age of all considered samples. In this version the plot shows values of centroid (median) and 50% and 95% confidence intervals of calendric age of all considered samples. Numerical values are available in form of a table.

As an illustrative example of this option we will consider calibration of the set of seventeen C-14 dates quoted by Hays (1984: 214) from Predynastic sites in El Khattara and Hierakonpolis. The results of "cut-out" calibration are shown in Fig. 5, while Fig. 6 shows results obtained using third version with indicated median values (dots), interquartile ranges (bold lines) and 95% confidence intervals of calendric age. Exact numeric data are listed in Table 1.

It seems that chronologic pictures presented in Figs. 5 and 6 are too sophisticated to lead to clear interpretation and definite conclusions. Dates obtained on samples from considered sites and cultures show significant scatter, the 50% and 95% confidence intervals overlap, covering interval from *ca.* 3,200 to 4,200 B.C. It can be noted, however, that conventional C-14 dates of each group quoted by Hays (1984: 214) are consistent with the only exception of date WSU-1730: $4,250 \pm 130$ B.P., which is *ca.* 500 years younger. Therefore, dates belonging to

Table 2

Concise calibrated radiocarbon chronology of Predynastic cultures and sites.

No.	Lab. code and no.	Age C-14 conv. B.P.	Median cal. B.C.	Interquartiles cal. B.C.	95% conf. int. cal. B.C.
1	Gerz ST	4701 ± 45	-3466	[-3519, -3414]	[-3614, -3374]
2	Amr HL14	4830 ± 120	-3617	[-3705, -3521]	[-3919, -3357]
3	Bad Ham	5218 ± 100	-4061	[-4150, -3985]	[-4305, -3812]
4	Bad HL11	4718 ± 66	-3498	[-3577, -3424]	[-3644, -3366]
5	Bad KH1	5038 ± 49	-3869	[-3913, -3809]	[-3964, -3719]
6	Bad KH3	4981 ± 77	-3794	[-3886, -3731]	[-3959, -3644]
7	Bad KH6	4921 ± 52	-3727	[-3762, -3690]	[-3902, -3628]

Hays 1984: 214, Table 1.

Gerz - Gerzean; Amr - Amratian; Bad - Badarian.

ST - South Town; HL14 - Hierakonpolis Loc. 14; Ham - Hamamiya; HL11 - Hierakonpolis Loc. 11; KH - Khattara.

same sites can be treated jointly, by the procedure of calibrating of mean value of several dates. The results, shown in Fig. 7 in the same form as in Fig. 6, give a clear insight into chronologic relations of considered cultures and their occurrence on excavated sites. Numeric data, listed in Table 2, can be interpreted as estimates of the duration of considered cultures.

Calibration of a set of related dates

This option is useful for calibration of groups of dates obtained on samples from definite site or culture. The result is presented as composite probability distribution of all dates in form similar to second screen produced by first op-

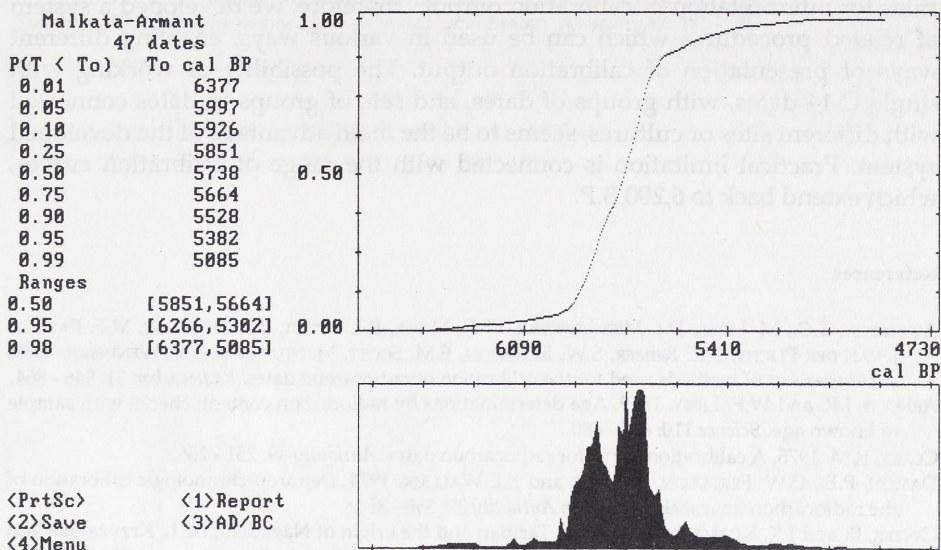


Fig. 8. Example of calibration output obtained with option for calibrating of a set of related dates (47 dates from site 21/83 in Malkata-Armant; cf. Ginter *et al.* 1985).

tion for calibration of a single date. An example of calibration output of a group of 47 dates obtained in the Gliwice Radiocarbon Laboratory on charcoal samples from Malkata-Armant (site 21/83) is shown in Fig. 8. The composite probability density function shows the presence of two well-distinguished peaks, which can be interpreted as resembling main phases of human activity on this site. The several seasons of excavation on this site, performed by the team directed by B. Ginter and J.K. Kozłowski, has led to distinguishing of three phases of occupation (Ginter *et al.* 1985). At that moment more than 20 samples from this region (including site 21/83) are being processed in the Laboratory, so the final conclusions cannot be stated.

Final remarks and conclusions

In the beginning of the radiocarbon dating the known-age Egyptian samples were used by W.F. Libby to test his idea. Now, after 40 years, almost all users of C-14 dates are aware of discrepancies between (conventional) C-14 time scale and calendric chronology. However, it seems that the need for comparing both C-14 and calendric chronology remains unchanged, and, moreover, such comparisons are of crucial importance for studies of the prehistory of Egypt and adjacent regions.

Described set of computer procedures is far from perfection, but, in spite of this, it enables presentation of results of C-14 dating in terms of calendric ages, using strict statistical concepts such as median, interquartile range, *etc.* The examples discussed in the text of this article show that there are no general rules for interpretation of calibration output. Therefore, we developed a system of related procedures which can be used in various ways, enabling different ways of presentation of calibration output. The possibility of working with single C-14 dates, with groups of dates, and sets of groups of dates connected with different sites or cultures, seems to be the main advantage of the developed system. Practical limitation is connected with the range of calibration curves, which extend back to 6,200 B.P.

References

- AITCHISON, T.C., M. LEESE, D.J. MICHCZYŃSKA, W.G. MOOK, R.L. OTLET, B.S. OTTAWAY, M.F. PAZDUR, J. VAN DER PLICHT, P.R. REIMER, S.W. ROBINSON, E.M. SCOTT, M. STUIVER and B. WENINGER. 1989. A comparison of methods used for the calibration of radiocarbon dates. *Radiocarbon* 31: 846 - 864.
- ARNOLD, J.R. and W.F. LIBBY. 1949. Age determinations by radiocarbon content: checks with sample of known age. *Science* 110: 678 - 680.
- CLARK, R.M. 1975. A calibration curve for radiocarbon dates. *Antiquity* 49: 251 - 266.
- DAMON, P.E., C.W. FERGUSON, A. LONG and E.I. WALLICK. 1974. Dendron-chronologic calibration of the radiocarbon time scale. *American Antiquity* 39: 350 - 366.
- GINTER, B. and J.K. KOZŁOWSKI. 1984. The Tarifian and the origin of Nagadian, In: L. Krzyżaniak and M. Kobusiewicz (eds.), *Origin and early development of food-producing cultures in North-Eastern Africa*: 247 - 260. Poznań: Polish Academy of Sciences, Poznań Branch, and Poznań Archaeological Museum.
- GINTER, B., J.K. KOZŁOWSKI and M. PAWLIKOWSKI. 1985. Field reports on the survey conducted in Upper Egypt in 1983. *Mitteilungen des Deutschen Archäologischen Instituts Abteilung Kairo* 41: 15 - 42.
- HASSAN, F.A. and S.W. ROBINSON. 1986. High-precision radiocarbon chronometry of ancient Egypt and comparisons with Nubia, Palestine and Mesopotamia. *Antiquity* 61: 119 - 135.
- HAYS, T.R. 1984. Predynastic development in Upper Egypt. In: L. Krzyżaniak and M. Kobusiewicz (eds.), *Origin and early development of food-producing cultures in North-Eastern Africa*: 211-220. Poznań: Polish Academy of Sciences, Poznań Branch, and Poznań Archaeological Museum.
- KRZYŻANIAK, L. and M. KOBUSIEWICZ (eds.). 1984. *Origin and early development of food-producing cultures in North-Eastern Africa*. Poznań: Polish Academy of Sciences, Poznań Branch, and Poznań Archaeological Museum.
- MICHCZYŃSKA, D.J., M.F. PAZDUR and A. WALANUS. 1990. Bayesian approach to probabilistic calibration of radiocarbon dates. *PACT* 29: 69 - 79.
- PAZDUR, M.F. and D.J. MICHCZYŃSKA. 1989. Improvement of the procedure for probabilistic calibration of radiocarbon dates. *Radiocarbon* 31: 824 - 832.

- PEARSON G.W., J.R. PILCHER, M.G.L. BAILLIE, D.M. CORBETT and F. QUA. 1986. High precision ^{14}C measurements of Irish oaks to show the natural variations from A.D. 1840 to 5210 B.C. *Radiocarbon* 28: 911 - 934.
- PEARSON, G.W. and M. STUIVER. 1986. High precision calibration of the radiocarbon time scale, 500 - 2,500 B.C. *Radiocarbon* 28: 839 - 862.
- RALPH, E.K., H.N. MICHAEL and M.C. HAN. 1973. Radiocarbon dates and reality. *MASCA Newsletter* 9: 1 - 9.
- ROBINSON, S.W. 1985. *A computational procedure for utilization of high-precision radiocarbon curves*. Open-File Report, USGS, Menlo Park.
- STUIVER, M. and G.W. PEARSON. 1986. High-precision calibration of the radiocarbon time-scale, A.D. 1,950 - 500 B.C. *Radiocarbon* 28: 805 - 838.
- Suess, H.E. 1970. Bristlecone pine calibration of the radiocarbon time scale 5,200 B.C. to the present. In: I.U. Olsson (ed.), *Radiocarbon variations and absolute chronology. Proceedings of the XII Nobel Symposium, Stockholm 1970*: 303 - 311. Stockholm: Almquist and Wiksell.
- Suess, H.E. 1979. A calibration table for conventional radiocarbon dates. In: R. Berger and H.E. Suess (eds.), *Radiocarbon dating. Proceedings of the 9th International Conference, Los Angeles and La Jolla, 1976*: 777 - 784. Berkeley-Los Angeles-London: University of California Press.
- SWITSUR, V.R. 1973. The radiocarbon calendar recalibrated. *Antiquity* 47: 131 - 137.