Perfection and Uncertainty

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ABSTRACT: Perfection has traditionally been associated with the Platonic ideals of form and beauty in the fields of aesthetics, art, sculpture and architecture. But in science and engineering applications perfection is also related to the limiting bounds of quality, and it is closely linked with uncertainty. When a copy or surrogate is so close to the reference or original that one cannot reliably measure or observe the difference, then it may be considered perfect. This paper explores dimensionality and appearance through three examples: (1) In 2D and 3D digitisation, how finely should a cultural object be sampled so that all the detail applied by human hand is captured? (2) In photogrammetric 3D coordinate measurement, what are the limiting factors on precision and accuracy? (3) In the reproduction of 2D images and 3D objects, how good should be the match of attributes such as tone, colour and gloss?

1. PERFECTION

The online edition of the *Oxford English Dictionary* gives several meanings for the noun 'perfection', among which are: (a) Consummation: the state of being completed; (b) Maturity: the most complete stage of growth or development of a person or thing; (c) Flawlessness: the condition of being free from defect; (d) Epitome: the ideal example of a person or thing; (e) Utmost quality: the most perfect performance or execution of a virtue, ceremony, office, etc. Thus the nuances of the word include not only a thing perfect within itself, but also something that is perceived to be the best possible in its form, behaviour or functionality.

These definitions can be traced back to Aristotle, who in Book *Delta* of the *Metaphysics* distinguished three different concepts: (1) completeness – contains all the requisite parts; (2) goodness – nothing of the kind could be better; and (3) fitness – has attained its purpose [1]. The parallel existence of two concepts of perfection, one strict and the other loose ('excellence'), has given rise since the Renaissance to a paradox: that the greatest perfection is imperfection, as formulated by Vanini (1585–1619). He argued that if the world were perfect, it could not improve and so could not attain 'true perfection' which depends on progress. Therefore perfection depends on incompleteness, since the latter possesses a potential for development and augmentation with new characteristics. It follows that the perfection of an art work lies in its forcing the recipient to be active, so to complement the work by an effort of mind and imagination.

Figure 1: *Plato and Aristotle, detail from the School of Athens, by Raphael (1509), Stanza della Segnatura, Pontifical Palace, Rome.*

The converse, that imperfection may be perfect, reveals an underlying psychological principle. Just as the so-called 'beauty spot' applied to an already beautiful face may make it appear more beautiful, so a flaw in an otherwise flawless work of art may enhance its apparent perfection. This applies not only to human vanity, but also to technology. Thus irregularity in semi-conductor crystals (an imperfection, in the form of contaminants) is an essential requirement for their semiconductivity. The paradox is resolved by distinguishing between perfect regularity and perfect utility [2].

Figure 2: *Platonic solids: tetrahedron, cube, octahedron, icosahedron, dodecahedron.*

Closely related to the concept of perfection is the notion that idealised forms may contain the essence of real forms. Plato proposed abstract archetypes that determine the form and function of manifested objects, images, symbols, or patterns. In 3D geometry the Platonic solids are regular, convex polyhedra, constructed by congruent polygonal faces with the same number of faces and edges meeting at each vertex (Fig. 2).

Plato's interpretation of universals was linked to his *Theory of Forms* in which he used both the terms εἶδος ('form') and ἰδέα ('characteristic'). Forms are abstract objects or παραδείγματα ('paradigms'), of which particular objects and their properties and relations are instances [3]. Beauty can likewise be idealised as an abstract quality, of which beautiful objects or shapes are instances. Within Plato's works, however, beauty is never subsumed within the good, the appropriate, or the beneficial. Beauty behaves as canonical Platonic Forms do, discovered through the same dialectic that brings other Forms to light. Nevertheless beauty is not just any form. Because human judgement of beauty is associated with aesthetic pleasure, comparison leads naturally to value scales. Plato often referred to harmony as a cause of beauty, and the perception of harmony means the perception of unity in variety, of the one in the many, which has been called "the one true aesthetic principle recognized by antiquity in general". Because to Plato harmony meant moderation, it was then an easy step from harmony to measure [4].

2. UNCERTAINTY

Plato's examples of what we might today call universals included mathematical and geometrical concepts such as natural numbers and the circle. Platonic form can be illustrated by contrasting a material triangle with an ideal triangle. The Platonic form is the ideal tetrahedron, a figure with perfectly drawn lines whose angles for the triangle on each face add to 180 degrees. Any real tetrahedron or triangle, however, will be an imperfect representation of the ideal. Regardless of how precise the measuring and drawing tools, it will never be possible to recreate the perfect shape. Even if constructed to a quality where our senses cannot perceive a defect, at some scale the shape will still be imperfect, unable to match the ideal.

Thus all real forms contain errors, or departures from the ideal. The straight line is never perfectly straight, the constructed circle is never perfectly circular, and so on. This is why the freehand drawing of a circle has classically been such a challenging exercise for artists, both a symbol of striving toward perfection [5] and a consummate test of skill (Fig. 3).

Figure 3: *Self-Portrait with Two Circles, by Rembrandt (1659), Kenwood House, London.*

The ideal can thus be regarded as a limit to which the real form approximates. Differences between the real and the ideal are always present at some scale. The principle of limits is at the heart of the calculus, where the differential gradient is approached as the limiting value of the ratio as the interval approaches zero, and the integral is the limiting value of summation over an infinite number of infinitesimally fine sections. Yet curiously some continuous functions are not integrable. The Lorentz distribution (or Cauchy distribution as the French prefer), which is the distribution of a random variable that is the ratio of two independent standard normal variables, is not integrable because its probability density function has no mean. Compared with the Gaussian function, it has a narrower peak but heavier tail and does not approach zero fast enough to allow the mean to converge (Fig. 4).

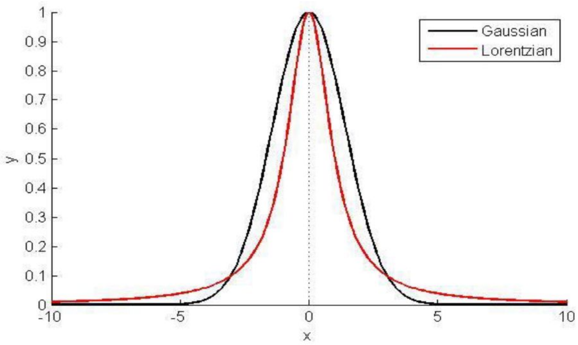

Figure 4: *Normalised Gaussian and Lorentzian distributions.*

Other contours and surfaces may not be measurable because the amount of detail depends on the scale at which they are viewed. In a forerunner to his theory of fractals, Mandelbrot examined the coastline paradox, the property that the measured length of a coastline depends on the scale of measurement [6]. Thus the smaller the increment, the longer the measured length becomes. If one were to measure a stretch of coastline with a 10- metre chain, one would get a shorter result than if the same stretch were measured with a one-metre rod, because the former would be laid along a less convoluted route than that followed by the latter. The logical consequence is that, if extrapolated, the measured length would increase without limit as the measurement scale is reduced towards zero.

Heisenberg famously expounded the uncertainty implicit in quantum theory. There is an inverse relationship between the certainty with which one can know both position and momentum. The more precise is one, the more imprecise the other [7]. In the Newtonian world the centre of mass of every particle can be specified precisely, but in the quantum world the particle is spread out like a probability distribution, so one cannot say exactly where the particle is, only the probability of its being at a particular place. The same applies with any pair of conjugate variables, such as time and frequency: a nonzero function and its Fourier transform cannot both be sharply localised.

Uncertainty in measurement may also arise from the so -called 'observer effect', wherein the very act of observing changes the state of what is being observed. This occurs not only at the quantum level but also at the macroscopic level. For example, in order to measure the total radiant power emitted in all directions from a source, one must integrate over a complete sphere. But how to measure the summation? Any sensor introduced into the integrating sphere will disturb the distribution of radiation; any porthole will reduce the amount collected.

Uncertainty affects all measurement tasks in the form of noise. In every electrical signal except at a temperature of absolute zero there is a noise component, caused by the vibration of the atoms in the conducting material. The random variation of thermal noise sets a minimum threshold for detection of the signal, i.e. a masking value below which the signal cannot be detected. The ratio of the maximum possible signal to this threshold is the signal-to-noise ratio (SNR), a fundamental parameter of any communication channel. For a sensor such as a CCD or CMOS array in a camera, which counts the number of photons incident on each cell, there is also Poisson noise, by the probability of a number of discrete events occurring in a fixed interval of time and space [8].

The consequence of noise sources in images from digital cameras is a random variation of the pixel values, even in regions that might be expected to be perfectly uniform. The MiniMacbeth colour target in Fig. 5 is carefully manufactured so that each patch is evenly coated with paint of a single colour. It was photographed on a copystand with uniform illumination, using a high quality 105mm macro lens on a Nikon D200 camera. Analysis of the pixel values along a cross-section, however, shows that they are not constant over each patch, but have considerable random variation. For the green patch the mean value is 13023 (in the 16-bit integer range 0-65535) and the standard deviation is 228.3, so the signal-to-noise ratio (SNR) is 57. This is surprisingly low for such a clearly defined feature, in a high quality image acquired under near-optimal conditions.

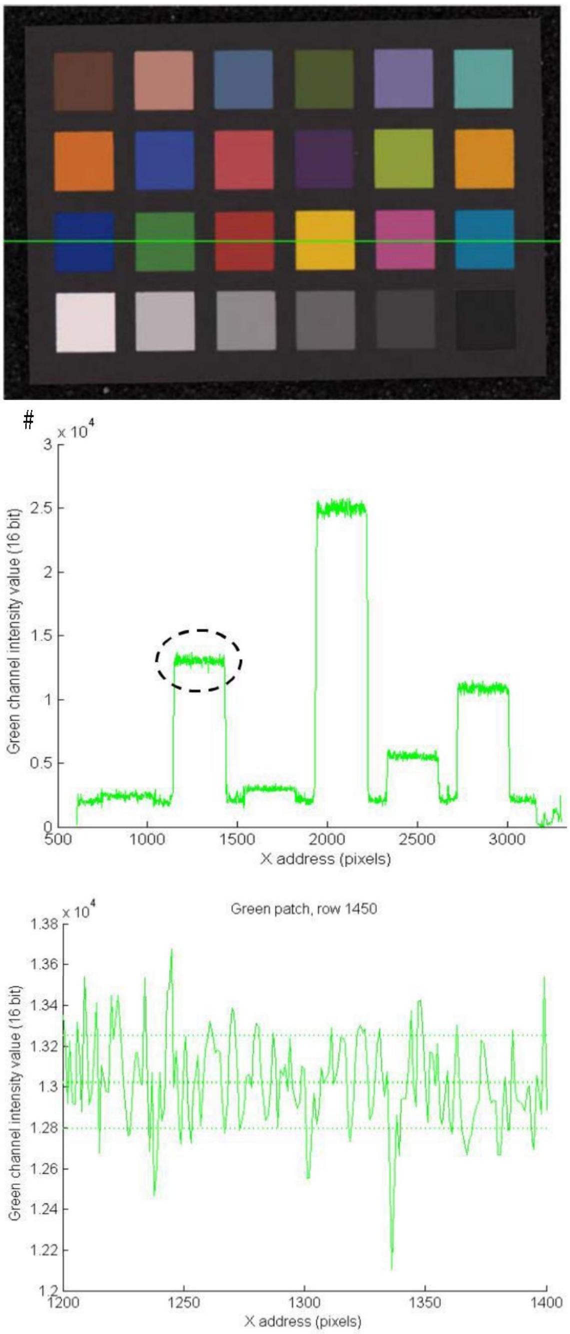

Figure 5: *Noise in a digital image: (top) Mini-Macbeth colour target with green line showing horizontal cross-section; (middle) elevation of intensity in green channel of image; (bottom) section of green patch with mean and stdev.*

Digital encoding introduces yet more noise by quantising the signal into a finite number of discrete steps, for example the integers 0-4095 in a 12-bit data format. The uncertainty is ±half of the interval between successive data values, so for example the digital value 412 serves to represent all analogue (continuous) values in the range 411.5 to 412.5.

Similar considerations apply to any quantised scale. The humble ruler with lines scribed at 1mm intervals has an intrinsic precision of ±0.5 mm. A skilful observer might be able to interpolate by eye to a precision of ±0.1 mm, but the uncertainty always remains. Repeated measurements of the same dimension by the same or different observers, using the same ruler, will therefore yield different values scattered within the range of uncertainty, following a normal distribution. From these the mean and standard deviation give the most likely value and a measure of the variation.

3. TEXTURE

If all objects in the world followed ideal forms, with perfect geometric shapes and perfectly smooth surfaces, the world would be a very boring place, at least from a visual viewpoint. Early computer graphics often evoked such environments, which invariably appeared artificial (Fig. 6).

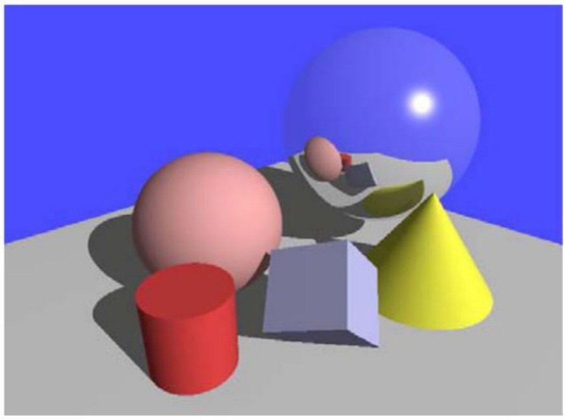

Figure 6: *Example of image created by ray tracing (Henrik Wann Jensen, 1990).*

The real world is not at all perfect, but full of irregular shapes and clutter. Paradoxically, the irregularity present in natural materials results in a more attractive appearance than uniform and flawless surfaces. Real surfaces are textured with many spatial frequencies, diverse patterns, and random variations. The human visual system has evolved to interpret the complex array of textures in each scene in order to recognise objects, obstacles and hazards in the surrounding environment, and thereby to facilitate wayfinding and navigation. Texture actually helps us to 'read' surfaces, to understand their materiality and hence their physical properties, such as weight, hardness, roughness, etc. The endless variety of real textures adds immeasurably to the stimulation of the visual system and consequent satisfaction gained from scanning over a scene.

The materiality of an object is one of the ways that it conveys its history: signs of toolmarks, decoration, and texture of the substrate show how it was made; signs of granularity, wear, damage, cracking, weathering and decay show its degradation with the passage of time (Fig. 7). All combine to give the impression that it is the 'real thing'. Even if one cannot touch the object and feel its tactile qualities, the patina of age somehow transforms its degraded nature into something evocative, even romantic, adding to its charm and visual power. The appearance of an old object is inseparable from its 'pastness' [9]. It follows that in any visualisation of a cultural heritage object the patina needs to be reproduced accurately, or at least in a way that is convincing. Plaster casts and 3D prints are invariably disappointing in this respect.

***Figure 7:** Censer, China, Zhejiang Province, Southern Song period, late 12th century stoneware with glaze showing crazing. Asia Society's Rockefeller Collection.*

4. IMPERCEPTIBILITY

In cognitive psychology a key quantity in psychophysical experiments is the 'just noticeable difference' (JND). This is the smallest change in the stimulus for which the probability of detection is 50%. Thus one subject might detect the change in half the trials, or one half of a population of subjects might detect it in all trials. Either way, it is indicative of the sensory threshold prevailing in the experimental conditions. The JND may be considered the midpoint of the psychometric function, which relates the probability of detection to stimulus intensity.

For many sensory phenomena, including brightness, loudness, pressure and pain, the magnitude of the detection threshold Δ turns out to be proportional to the magnitude of the stimulus . Thus one can write:

Δ

This behaviour applies to all the senses, and is known as the Weber ratio. It follows that perceptual response to sensory stimulus is logarithmic. Fechner used this law to construct a psychophysical scale relating the physical magnitude of a stimulus and its (subjectively) perceived intensity [10]. A good example is the darkness scale used in photographic exposure, which is arranged in equal increments of photographic density. Thus a transmission of 10% of incident light is density 1.0, of 1% is density 2.0, etc. Although very non-uniform in terms of reflectance factor, the scale appears rather uniform (Fig. 8).

***Figure 8:** Kodak photographic density scale.*

Thus the JND is not constant in absolute magnitude but varies throughout the perceptual gamut. It is important in colour judgements, for example, when deciding whether a retouching paint is sufficiently close to the original pigment colour. Much experimental research has sought to establish a perceptually uniform colour space, in which one JND corresponds everywhere to one unit of distance between the coordinate points. This turns out to be a difficult problem because it depends on the spectrum of the illumination, adaptation to the viewing field and differing sensitivity between observers. The CIE system of colorimetry has attempted to eliminate the sources of variability through a 'standard observer', with a defined spectrum of daylight (most commonly D65), and a defined colour space (CIELAB) with an associated Euclidean colour difference metric (ΔE^*). More recent models (CIECAM) perform better.

What is the finest spatial detail that can be rendered onto the surface of an artefact by a human craftsman? Study of real objects in various media, as well as the contrast sensitivity of human vision, suggests a dimension of approximately 40 µm [11]. It follows that if one can reproduce decorative surface detail with a spatial resolution of at least 50 points/mm, or about 1250 dots per inch (dpi), then it should be indistinguishable from the original. This is a useful guideline for the fineness of sampling during digitisation, as well as in printing.

5. VERIDICALITY

Veridical means a close match to a well-accepted norm or standard. If a veridical reproduction were placed alongside the original, ideally one could not tell them apart. A perfect reproduction would in every way be a facsimile of the original, and would faithfully reproduce all its defects and flaws. Moreover every attribute could be measured and metrics could be applied against quantitative criteria, such as dimensions, shape, colour, weight, gloss, etc. With great care and control of the reproduction parameters it is possible to achieve veridicality in the reproduction of canvas paintings when viewed side-by-side in a cabinet or on the gallery wall (Fig. 9). Even this is usually only a colorimetric reproduction, however, in which the combination of colorants at each point is a metamer of the colorants in the original relative to the prevailing light source. Under a different illumination spectrum the two pictures might appear different [12].

Figure 9: *The rare situation of assessing two printed reproductions side -by-side with the original painting in situ in the gallery. 'Adoration of the Kings', by the Master of Liesborn c.1475, in the National Gallery London. MARC project, 1993.*

The alternative is 'pleasing reproduction', producing something that looks good. It may not be a perfect match with the original in a metric sense, because defects may have been removed and attributes enhanced. When assessed in isolation by a panel of observers on multiple visual quality scales, however, such a reproduction will often be preferred and ranked more highly than a veridical reproduction. Examples are to be found in every museum shop, where the customer sees only the facsimile in isolation and cannot make a side-by-side comparison with the original, so the makers are inclined to render the reproduction in such a way to make it more attractive.

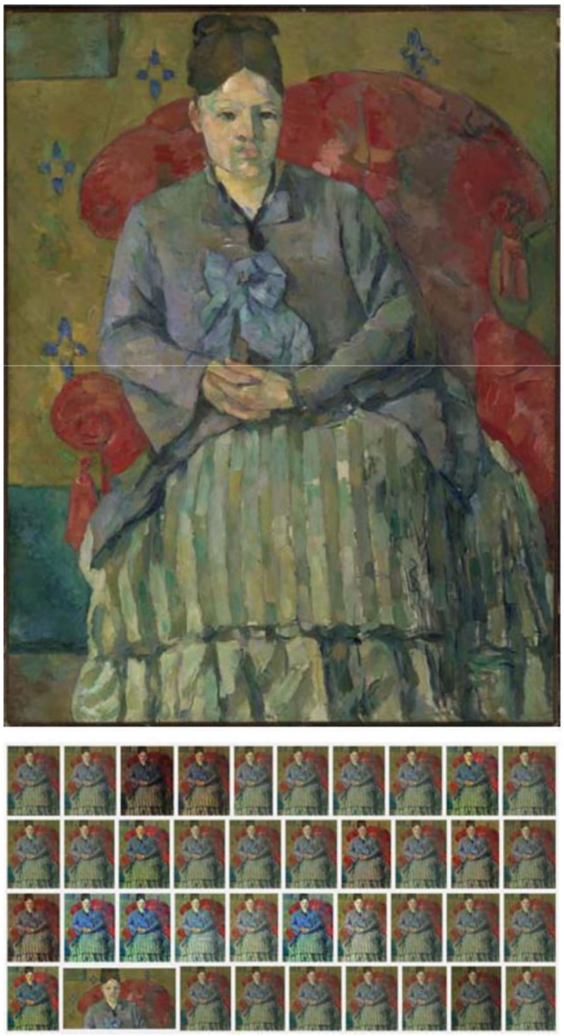

Figure 10: *(top) 'Madame Cézanne in a Red Armchair' by Cézanne, 1877, Museum of Fine Arts, Boston (bottom) A selection of internet images resulting from a Google search.*

Yet if each person reproducing an artwork exercises a personal rendering preference, then not only will each reproduction differ from the perfect canonical reproduction, but they will all differ from one another. In the presence of such a diversity of possibilities, how is one to know which is correct, or indeed which is closest to the actual painting (Fig. 10)? Moreover what if the painting has faded or discoloured or is covered with dark varnish? Which of the many reproductions is really like the original as produced by the artist and viewed in the light of his studio? Cézanne had north- facing daylight through the enlarged windows in the upstairs studio of his house near Aix- en-Provence, but today we rarely view images under such ideal conditions, with little control over illumination.

6. DIMENSIONALITY

Regression techniques enable an ideal form, such as a line or curve or plane or sphere, to be fitted to a measured set of points perturbed by noise. The overall error between the ideal and the actual points is minimised according to some measure such as the sum of the squares of the differences between nearest points. Such procedures are indispensable in every field of science and engineering (Fig. 11).

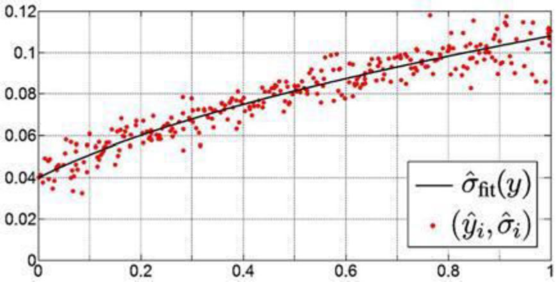

Figure 11: *Fitting of a curve through scattered experimental data (from Foi et al [8]).*

Fitting in three dimensions has become an important issue in the processing of point clouds generated by 3D scanners. Just as the 2D image of a uniformly coloured surface has inherent noise (Fig. 5), so the 3D point cloud of a planar surface has an intrinsic irregularity, arising from physical noise processes (Fig. 12).

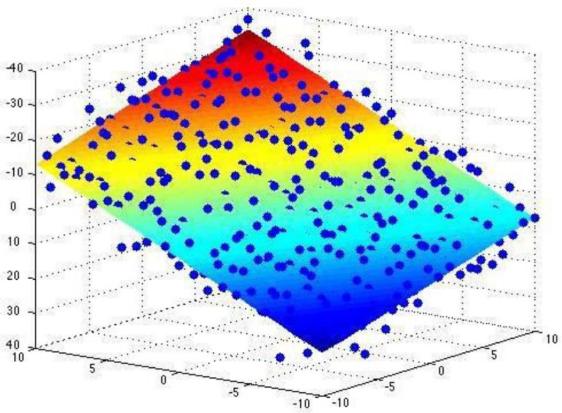

Figure 12: *Fitting of a plane through scattered points in a 3D point cloud (from Mathworks).*

The problem is exacerbated when multiple point clouds of the same surface, taken from differing viewpoints, have to be merged together. Because of noise and the effects of transparency and specularity, there is never a perfect correspondence and so 3D coordinates have to be estimated by some process that takes into account the positions of all the points in the neighbourhood. The residual uncertainty in the approximation cannot be avoided.

7. AUTHENTICITY

Perception of difference raises questions that are fundamental to making reproductions of art works. If the surrogate is so close to the original that human perception cannot distinguish one from the other then can it be said to be identical? Does it matter if the surrogate cannot be discriminated from the original? The answer depends on the purpose of making the copy: a replica is legitimate whereas a counterfeit is fraudulent. The value placed on the object depends more on its authenticity than on the characteristics of the object itself. Hence the importance of provenance, tracing ownership back in an unbroken chain to the artist.

There are good reasons for making veridical copies of artworks, of course. If an object is too fragile or too valuable, a museum may choose to send a copy to an exhibition elsewhere. Does this substitution need to be made known to the viewing public? Is the experience of looking at an artwork or ancient artefact equally valid if the observer believes that he or she is viewing the real thing? Does the significance of the object lie in its authenticity or in its appearance or in its narratives and associations?

The issue of authenticity is most critical in the art market, where the monetary value of an object depends on establishing an unequivocal link to the artist. Enter the *catalogue raisonné*, the comprehensive and authoritative listing of all known works by the artist. Inclusion of a work can increase its price by a factor of 100. For example, in a recent case a painting by the French post-impressionist Édouard Vuillard became worth approx. £250,000 when accepted in 2014 for inclusion by the Wildenstein Institute in the catalogue raisonné, against only £1,500 "as a piece of decorative art" [13].

Figure 13: *'The Oysters' now attributed to Vuillard, 1918.*

Forgery is much more problematic for antiquities, where the artist is unknown and provenance *ab initio* cannot be established [14].

8. REALITY

Ultimately perfection is an ideal that will always be unattainable. In every real object, and in every digital representation of it, there lies imperfection caused by random variation or inaccuracy at some scale. Yet the perfectionist is never satisfied, but always wants something beyond what exists. The meaning of Voltaire's aphorism "The perfect is the enemy of the good" [15] goes beyond project management to fundamental human psychology. In the real world ubiquitous uncertainty sets the limits.

Richard Gregory made a case study of a man ('S.B.') blind from birth, whose sight had been restored at the age of 52 by an operation [16]. When blind he had learned the characteristics of objects by touch and could describe them intelligently. Yet after regaining his sight he found great difficulty in reconciling the shapes he saw as projections of solid forms, and his visual perception seemed unable to apply size constancy to compensate for distance. "He also found some things he loved ugly (including his wife and himself), and he was frequently upset by the blemishes and imperfections of the visible world." The patient became irreconcilably depressed and soon died.

Oliver Sacks [17] related the story of another man ('Virgil') whose sight was restored at the age of 50. With both eyes working, he went back to work as a masseur, but found that much of his visual world was confusing. "Although he thought he knew all the bodies of his clients, now he found himself startled by seeing bodies, and skins, that he had previously known only by touch; he was amazed at the range of skin colours he saw, and slightly disgusted by blemishes and 'stains' in skins that to his hands had seemed perfectly smooth. He found it a relief, when giving massages, to shut his eyes." Virgil was increasingly unable to cope, then fell ill and succumbed to pneumonia.

In these cases, and others reported in the literature, the person during a lifetime of blindness had learned to perceive the world as consisting of ideal shapes populating an ideal three dimensional space. But with sight had come the revelation of the imperfect forms of objects, the mottling and textures of their surfaces, their irregularity and unpredictable motion. Thus perfection is revealed to exist only in the imagination, while the real world is laden with imperfection and uncertainty.

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